PREFACE

I am very happy to present this student friendly text book ***ENGINEERING MATHEMATICS-I*** *on this auspicious day of Vinayaka Chathurthi. This self explanatory comprehensive book is in accordance with the VTU syllabus w.e.f the year 2010-11. It also caters to the need of autonomous institutions in Karnataka and other technological universities in the country. Student requirement and aspirations are the prime factors kept in mind in the compilation of this book in a lucid way.*

Two of my conscious senior colleagues in the department ***Dr. D. Mamta*** *and* ***Ms. G. V. Pankaja*** *have shouldered the responsibility in my effort to make this book error free. I am highly thankful to them in this regard.*

I am very much indebted to ***Mr. K.V. Balakrishna*** *of* ***M/s. Sudha Publications*** *for the immense confidence in my authorship, continuously publishing my books for the last 14 years accepting all my suggestions.*

Systematic approach by way of computer type setting work racing with the time by ***Sri. S. Raghunandan*** *and his team of* ***M/s. Allkind*** *is highly commendable.*

I thank the printers for the quality offset printing.

I am confident that the esteemed readers will bestow the same kind of response as in the past. I will humbly receive all the comments and valuable suggestions from the community of readers.

September 11th 2010 **K.S.Chandrashekar**

Mysore - 8

REWARD

VTU students of the current scheme w.e.f 2010-11, scoring 125/125 in all four papers of Engineering Mathematics I to IV Semesters (10 MAT 11, 21, 31, 41) will be rewarded with a cash prize of Rs.7,500/- by the author. Please write to the author directly along with attested xerox copies of marks cards of all the four semesters.

Achiever: ***Mr. Bharath M.V,*** **a student from 2006 batch of E&C branch from PESIT, Bangalore, received cash prize during 2008.**

SYLLABUS

ENGINEERING MATHEMATICS - I

Code	**: 10 MAT 11**	**IA Marks**	**: 25**
Hrs/Week	**: 04**	**Exam Hrs**	**: 03**
Total Hrs	**: 52**	**Exam Marks**	**: 100**

PART - A

Unit - I : Differential Calculus - 1

Determination of n^{th} derivative of standard functions (Illustrative examples). Leibnitz's theorem (without proof) and problems.

Rolle's theorem - Geometrical interpretation. Lagrange's and Cauchy's mean value theorems. Taylor's and Maclaurin's-series expansions of functions of one variable (without proof). **[6 hours]**

Unit - II : Differential Calculus - 2

Indeterminate forms - L'Hospital's rule (without proof), Polar curves : Angle between polar curves; Pedal equation for polar cuves. Derivative of arc length - concept and formulae without proof. Radius of curvature - Cartesian, parametric, polar and pedal forms. **[7 hours]**

Unit - III : Differential Calculus - 3

Partial differentiation : Partial derivatives, total derivative and chain rule. Jacobians - direct evaluation.

Taylor's expansion of a function of two variables-Illustrative examples. Maxima and Minima for function of two variables, Applications - Errors and Approximations.

[6 hours]

Unit - IV : Vector Calculus

Scalar and vector point functions - Gradient, Divergence, Curl, Laplacian, Solenoidal and Irrotational vectors. Vector Identities : div ($\phi \vec{A}$), Curl ($\phi \vec{A}$), Curl (*grad* ϕ), div (Curl $\vec{A}$), div ($\vec{A} \times \vec{B}$) and Curl (Curl $\vec{A}$).

Orthogonal Curvilinear Coordinates - Definition, unit vectors, scale factors, orthogonality of cylindrical and spherical systems. Expression for Gradient, Divergence, Curl, Laplacian in an orthogonal system and also in cartesian, cylindrical and spherical Systems as particular cases - No problems. **[7 hours]**

PART - B

Unit-V : Integral Calculus

Differentiation under the integral sign-simple problems with constant limits. Reduction formulae for the integrals of $\sin^n x$, $\cos^n x$, $\sin^m x \cos^n x$ and evaluation of these
integrals with standard limits - problems.

Tracing of curves in cartesian, parametric and polar forms - Illustrative examples. Applications - Area, Perimeter, Surface area and Volume. Computation of these in respect of the curves - (i) Astroid : $x^{2/3}+y^{2/3} = a^{2/3}$ (ii) Cycloid : $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ (iii) Cardioide : $r = a(1+\cos\theta)$

[6 hours]

Unit-VI : Differential Equations

Solution of first order and first degree equations : Recapitulation of the method of separation of variables with illustrative examples. Homogeneous, Exact, Linear equations and reducible to these forms, Applications : orthogonal trajectories.

[7 hours]

Unit-VII : Linear Algebra - 1

Recapitulation of Matrix Theory. Elementary transformations, Reduction of the given matrix to echelon and normal forms, Rank of a matrix, consistency of a system of linear equations and solution. Solution of a system of linear homogeneous equations (trivial and non-trivial solutions). Solution of a system of non- homogeneous equations by Gauss elimination and Gauss-Jordan methods. **[6 hours]**

Unit-VIII : Linear Algebra - 2

Linear transformations. Eigen values and eigen vectors of a square matrix, similarity of matrices, Reduction to diagonal form, Quadratic forms, Reduction of quadratic form into canonical form, Nature of quadratic forms. **[7 hours]**

Note : In the case of Illustrative Examples, questions are not to be set.

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